

A physics-based minimal model for perforation of a plate by a spherical projectile

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Abstract

In aerospace applications, ballistic limit equations (BLEs) are empirical correlations that give the largest projectile a protective target (shield) can stop. The NASA single-wall BLE relates the critical projectile diameter to the target thickness, hardness, density ratio, and impact velocity through power-law. As part of a high-school research project, we are interested in finding out if and to what extent that structure can be understood or replicated from basic physics. The problem is approached by considering two limiting idealizations of the target response, Hertzian indentation of a half-space and bending of a thin plate. Using energy balance, a geometric perforation criterion, and point-source description of impact cratering, a power law form for the critical diameter with density and velocity exponents is obtained for both idealizations, which is structurally comparable to the NASA BLE. Illustrative calculations are performed to compare and contrast the results from the models with those from the empirical BLE and experiments. We hope that this work can be a useful example of how familiar high-school physics and intermediate mechanics could help interpret an important empirical engineering equation.

Keywords: ballistic-limit equation (BLE); scaling; Hertzian indentation; plate bending; impact cratering; mechanics education

I Introduction

There are abundant examples of small objects impacting large (compared to the object size) surfaces. They may occur on aero-engine turbine blades as particles of sand or ice may be ingested ([Johns, 1974](#); [Chen, 2005](#)) at speeds on the order of 10^2 m/s. At even higher speeds ($\gtrsim \mathcal{O}(10^3)$ m/s), micrometeoroids and orbital debris (MMOD) impacts may occur in the lower Earth orbit, which may have serious

implications for spacecraft and satellite safety (Kessler and Cour-Palais, 1978). The design of structures against such high-velocity and hypervelocity impacts remains a topic of active interest (Whipple, 1947; Cour-Palais and Crews, 1990; Rogers et al., 2024). Empirical penetration relations (Cour-Palais, 1987), dimensional scaling (Clough et al., 1970; Watts et al., 1993; Watts and Atkinson, 1995; Holsapple and Schmidt, 1987), and experiments (Clough et al., 1966, 1970; Hörz et al., 1995) provide the background for engineering design equations (Christiansen et al., 2009; Ryan and Christiansen, 2010).

Against the rich background of the literature on projectile impact, cratering, and target perforation, the work presented in this article is limited in scope and focus. It presents the result of work that started as a high-school research project by the first author whose interest in materials science and space led to initial ideas that aimed to gain insight into the mechanics of a basic Whipple shield (Whipple, 1947). In the process, the authors engaged in a one-year exploration that started with basic high-school physics, meandered through Hertzian contact, fracture mechanics, and the mechanics of flexible plate structures. The direction was guided by the NASA ballistic limit equation (BLE)* (Christiansen et al., 2009), which gives the critical particle/projectile size (d_c) that results in perforation of a target plate of thickness t_t

$$d_c = \left[\frac{t_t \tilde{H}_t^{0.25} (\rho_t/\rho_p)^m}{5.24 k (v_p \cos \theta / c_t)^{2/3}} \right]^{18/19} \quad (1)$$

where $\tilde{H}_t$ is the Brinell hardness number, ρ the mass density, c_t the target wave speed, v_p the projectile velocity, and θ the impact angle with respect to the normal. The damage parameter k equals 1.8, 2.2, or 3.0 for perforation, detached spall, or incipient attached spall, respectively. The density exponent is $m = 0.5$ for $\rho_t/\rho_p > 0.67$ and $m = 2/3$ for $\rho_t/\rho_p \lesssim 0.67$. This empirical equation uses d_c and t_t in cm, densities in g/cm³, and speeds in km/s; its numerical coefficient is tied to these units.

For normal impact ($\theta = 0$), Eq. (1) can be written as

$$d_c = \left(\frac{t_t}{5.24 k} \right)^\alpha \tilde{H}_t^\beta \left(\frac{\rho_t}{\rho_p} \right)^\gamma \left(\frac{c_t}{v_p} \right)^\lambda \quad (2)$$

where $\alpha = 18/19 \approx 0.947$, $\beta = 9/38 \approx 0.237$, $\gamma = 9/19 \approx 0.474$ for $m = 0.5$, and $\lambda = 12/19 \approx 0.632$.

We do not develop a new BLE nor do we plunge into the theory of hypervelocity impact. Instead, we pose the following question: *how much of the power-law structure of the NASA BLE follows from basic high-school physics and undergraduate mechanics?* In other words, we seek a physics-based minimal model for the critical projectile diameter that will defeat (perforate) the target. Attention is limited to normal impact.

The layout of this paper is as follows. Section 2 develops the two limiting models. Section 3 introduces a strength correction based on impact cratering. Section 4 compares the models with the NASA BLE and some experiments, and Section 5 provides concluding remarks.

*Variables with subscript ‘p’ and ‘t’ indicate projectile and target quantities, respectively.

2 Our Approach

We view this problem from two vantage points that approximate the projectile–target interaction. In the first approach, we model it as Hertzian contact. In the second approach, we model it as a thin plate subjected to a point force. In both approaches, we adopt a geometric condition to characterize perforation of the target.

The kinetic energy of a spherical projectile of diameter d , density ρ_p , and traveling at a speed v_p is

$$\mathcal{K}_p = \frac{\pi}{12} \rho_p d^3 v_p^2 \quad (3)$$

The target is a plate of thickness t_t , density ρ_t , Young’s modulus E_t , and elastic wave speed estimated by the one-dimensional bar value $c_t = (E_t/\rho_t)^{1/2}$. For both idealizations, we assume that some fraction $\eta (\lesssim 1)$ of $\mathcal{K}_p$ is stored as strain energy (i.e., stored work).

2.1 Model I: elastic indentation

If the target resists locally, like a half-space indented by a spherical indenter of radius $R = d/2$, Hertz theory gives the contact force and stored work ([Tabor, 2000](#))

$$P(\delta) = \frac{4}{3} E^* \sqrt{R} \delta^{3/2}, \quad W_{\text{ind}} = \int_0^\delta P \, d\delta' = \frac{8}{15} E^* \sqrt{R} \delta^{5/2} \quad (4)$$

where δ is the indentation depth and E^* the effective contact modulus, simplified here to $E^* \approx E_t$. Equating $\eta \mathcal{K}_p = W_{\text{ind}}$ yields

$$\delta = K_i d \left(\frac{\rho_p v_p^2}{E_t} \right)^{2/5}, \quad K_i = \left(\frac{5\sqrt{2}\pi}{32} \right)^{2/5} \eta^{2/5} \approx 0.864 \eta^{2/5} \quad (5)$$

Eq. (5) shows that the indentation depth scales with the projectile size, and is moderated by the ratio of the so-called stagnation pressure ($\rho_p v_p^2$) of the projectile to the target stiffness (E_t).

Motivated by [Cour-Palais \(1987\)](#), we propose $\delta = t_t$ as the geometric perforation threshold. Then, using $E_t = \rho_t c_t^2$, the critical diameter that perforates the target is obtained as

$$d_c^{(1)} = \frac{t_t}{K_i} \left(\frac{\rho_t}{\rho_p} \right)^{0.4} \left(\frac{c_t}{v_p} \right)^{0.8} \quad (6)$$

We note that the elastic half-space idealization is at odds with the finite thickness of the target. More sophisticated formulations that account for the thickness effect in the indentation response are available ([Li et al., 2018](#)) but not pursued here to retain simplicity.

2.2 Model II: elastic plate bending

In the opposite limit, we consider the case where the target resists the projectile-induced force by bending over a characteristic span ℓ . For a transverse load P applied over a small region d (the indenter size) compared to ℓ (i.e., $q \equiv d/\ell \rightarrow 0$, see (Li et al., 2018)), the maximum static deflection scales as $w_{\max} \sim P\ell^2/D$ (Timoshenko and Woinowsky-Krieger, 1959), where $D = E_t t_t^3/[12(1 - \nu_t^2)]$ and ν_t is Poisson's ratio. The resulting stored bending energy scales as

$$\mathcal{U}_b = \frac{1}{2} P w_{\max} \sim C_b \frac{E_t t_t^3 w_{\max}^2}{\ell^2}$$

where C_b collects the geometric factors and the Poisson's ratio. For the problem at hand, the actual target span is supported far away ($\ell/t_t \rightarrow \infty$), which causes analytical difficulty. To mitigate this issue, we provide a somewhat hand-waving argument as follows. We posit that the actual span of the target plate that participates in bending is set by how far the impact-induced wave will propagate in the plane of the target over the loading time-scale, $\tau \sim t_t/v_p$. Then, dimensional analysis gives $\ell \sim (c_t t_t \tau)^{1/2} \sim \chi t_t$. For simplicity, we assume $\chi \sim \mathcal{O}(10^0)$ to be a constant.

The proposed condition for the perforation, $w_{\max} = t_t$, is motivated by the deformation within this local region. In the small-slope bending description, the characteristic curvature κ and the magnitude of the surface bending-strain ε_b scale as (Timoshenko and Woinowsky-Krieger, 1959)

$$\kappa \sim \frac{w_{\max}}{\ell^2}, \quad \varepsilon_b \sim \frac{t_t \kappa}{2} \sim \frac{t_t w_{\max}}{2\ell^2}$$

At $w_{\max} = t_t$ and $\ell = \chi t_t$, this gives $\varepsilon_b \sim 1/(2\chi^2)$, up to a factor associated with the deflected shape. A thickness-scale deflection over a short span can, therefore, involve substantial local deformation. We use it as an approximate failure condition for the locally deformed region. Equating $\eta \mathcal{K}_p = \mathcal{U}_b$ at $w_{\max} = t_t$ gives (with $\ell = \chi t_t$)[†],

$$d_c^{(2)} = \frac{t_t}{K_b} \left(\frac{\rho_t}{\rho_p} \right)^{1/3} \left(\frac{c_t}{v_p} \right)^{2/3} \quad (7)$$

where K_b collects η , χ , and the plate constants.

2.3 Comparison of limiting scalings

Table 1 collates the exponents from the two models along with those of the NASA BLE. In both models, the two exponents are related by $\lambda = 2\gamma$. This is because, in the energy balance, the information about the target's material property comes through E_t , and the information about the projectile through its density and velocity. By way of consequence, the ratio d_c/t_t can only depend on the group $E_t/(\rho_p v_p^2)$. With $E_t = \rho_t c_t^2$, the target material density enters Eq. (6) and Eq. (7) through the wave speed. On the contrary, the NASA BLE gives $\lambda = (4/3)\gamma$ (empirically so!). Notwithstanding, the functional forms of the models are encouragingly similar to the NASA BLE, save for the missing hardness term in the models.

[†]The exponents obtained in this idealization are identical to those used by Clough et al. (1970)

The NASA BLE density exponent is closer to the indentation value, whereas its velocity exponent is closer to the bending value. The thickness exponent is close to unity in all three expressions. These similarities motivate the comparison, although they do not unequivocally indicate if either, and if so, which, deformation mode controls perforation.

Table 1: Comparison of scaling exponents for the critical diameter.

Model	α	γ	λ
I: indentation	1	2/5	4/5
II: plate bending	1	1/3	2/3
NASA BLE, Eq. (2)	18/19	9/19	12/19

The hardness term accounts for the material strength, which is absent from the elastic calculations. In what follows, we introduce a correction factor to account for this unmodeled effect.

3 The correction factor

As noted earlier, a real target is neither an elastic half-space nor an ideal elastic plate subjected to a concentrated force. Non-linear and inelastic effects (e.g., plasticity, damage) are expected, in addition to other complicating effects (e.g., heating, softening) due to wave propagation during the impact process, which should result in a more compliant behavior than what is predicted by purely elastic behaviors. Our approach to account for these unmodeled effects and associated nonlinearities is to introduce a correction factor Φ_c such that the modified critical diameters (Eqs. (6) and (7)) can be succinctly expressed as

$$d_c^{(i)} = \frac{t_t}{\Phi_c} \left(\frac{\rho_t}{\rho_p} \right)^\gamma \left(\frac{c_t}{v_p} \right)^\lambda, \quad (i = 1, 2) \quad (8)$$

where the constants K_i and K_b in Eqs. (6) and (7) are absorbed into Φ_c . The thickness exponent is unity in both models, while γ and λ take the values in Table 1.

To obtain a functional form for Φ_c , we appeal to the mechanics of impact cratering. The point-source description of [Holsapple and Schmidt \(1987\)](#) offers a useful direction to account for the role of material strength. When impact occurs over a small region and a short time compared with the response of interest, the response away from the source can be described without resolving the details of the projectile. The projectile momentum and kinetic energy scale as $\mathcal{I} \sim \rho_p d^3 v_p$ and $\mathcal{K} \sim \rho_p d^3 v_p^2$. Their cube roots therefore suggest the simplified coupling measure

$$\mathcal{C} = \rho_p^{1/3} d v_p^n, \quad \frac{1}{3} \leq n \leq \frac{2}{3}, \quad (9)$$

where the lower and upper values of n correspond to momentum- and energy-controlled responses, respectively ([Holsapple, 1993](#)).

In the target, we estimate the dynamic stress at radial distance r from impact by $\sigma_{\text{dyn}}(r) = \frac{1}{2}\rho_t[v_t(r)]^2$ (Watts and Atkinson, 1995), with the particle velocity given by (Holsapple and Schmidt, 1987)

$$v_t(r) = C_u v_p \left(\frac{r}{d}\right)^{-1/n} \quad (10)$$

where C_u is a constant. Let L denote the distance at which this stress reaches an assumed threshold strength $Y_t = \alpha_Y H_t$. We use $\alpha_Y \approx 1/3$, as suggested by the relation between hardness and strength for metals (Tabor, 2000). The hardness H_t must be expressed in stress units in this balance (1 BHN $\simeq$ 9.81 MPa). Then

$$\frac{1}{2}\rho_t C_u^2 v_p^2 \left(\frac{L}{d}\right)^{-2/n} = \alpha_Y H_t, \quad (11)$$

which gives

$$\frac{L}{d} = \left(\frac{C_u^2 \rho_t v_p^2}{2\alpha_Y H_t}\right)^{n/2} \quad (12)$$

Thus, all other quantities remaining fixed, $L \propto H_t^{-n/2}$. We posit that the correction factor introduced in Eq. (8) follows the same dependence on the target hardness:

$$\Phi_c = \left(\frac{H_t}{H^*}\right)^{-\beta}, \quad \beta = \frac{n}{2} \in \left[\frac{1}{6}, \frac{1}{3}\right] \quad (13)$$

Here H^* is a constant with the units of hardness but it also embeds the constants absorbed into Φ_c . We take the midpoint $n = 1/2$, giving $\beta = 1/4$. Substituting Eq. (13) into Eq. (8) yields

$$d_c^{(i)} = \frac{t_t}{(H^*)^\beta} H_t^\beta \left(\frac{\rho_t}{\rho_p}\right)^\gamma \left(\frac{c_t}{v_p}\right)^\lambda \quad (14)$$

where $\beta = 0.25$, and γ and λ take the values for the two models in Table 1. A softer target has a larger correction factor and admits a smaller critical particle. The NASA BLE hardness exponent, $9/38 \approx 0.237$, lies close to the midpoint value. Section A explains how we calibrate H^* to the NASA BLE.

4 Results

4.1 Comparison with NASA BLE

We compare Eqs. (1) and (14) for an aluminum projectile ($\rho_p = 2700 \text{ kg/m}^3$) and three representative targets: Al 6061-T6, Ti-6Al-4V, and 304L stainless steel (Table 2). We evaluate Eq. (1) with each set of properties to compare the models on a common basis.[‡] The inputs are illustrative values retained from

[‡]The metallic NASA BLEs share the Cour-Palais form. For steel, hardness and wave speed are absorbed into a material coefficient. The expression for Ti in Christiansen et al. (2009, Eq. (4-8)) also changes the outer power from 18/19 to 1. Here Eq. (2) is used throughout; the comparison does not reproduce each material-specific calibration. See also Ryan and Christiansen (2010, pp. 9–10).

the original project. The three targets have nearly equal bar wave speeds while differing in density and hardness.

Table 2: Material parameters used in the comparison.

Target (s)	$\tilde{H}_t$ (BHN)	ρ_t (kg/m ³)	E_t (GPa)	c_t (m/s)
Al 6061-T6	95	2700	68.9	5052
Ti-6Al-4V	334	4420	114.5	5090
304L steel	215	7900	200.0	5032

We set $k = 1.8$ and $t_t = 5$ mm, and use 500 equally spaced velocities from 2 to 8 km/s. For these wave speeds, the interval corresponds approximately to $0.4 \leq v_p/c_t \leq 1.6$. A single value of H^* is fitted to both models and all three systems using the procedure in Section A. This gives $H^* \approx 4830$ BHN (for $t_t = 5$ mm and $k = 1.8$). We note that, in theory, the so-obtained H^* is not a material property, because it depends on the target thickness and k . In practice, however, the sensitivity is somewhat modest; for example, doubling the thickness increases H^* by $2^{4/19} \approx 16\%$, but since H^* enters Eq. (14) only as $(H^*)^{-1/4}$, the fitted d_c/t_t changes by just $2^{1/19} \approx 3.7\%$.

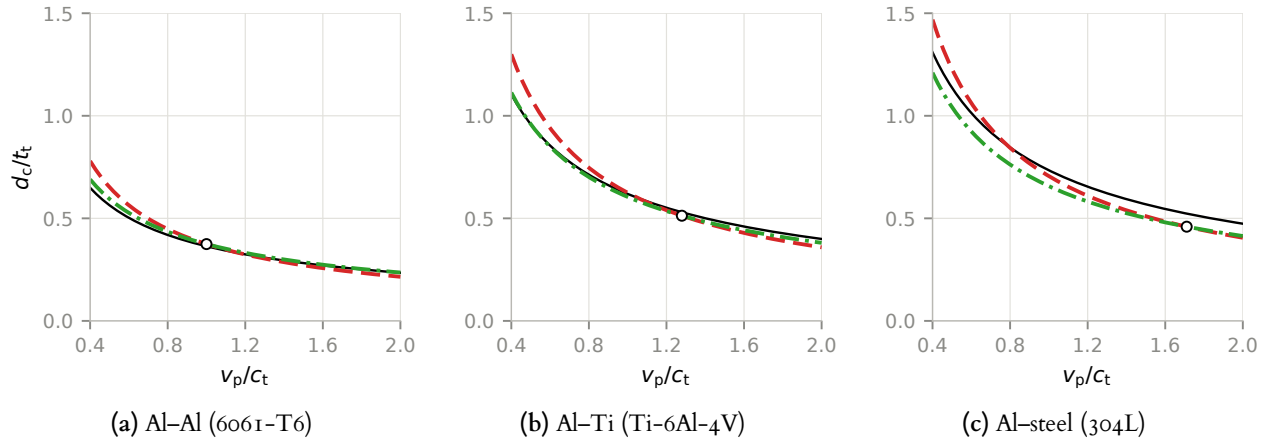


Figure 1: Normalized critical diameter versus normalized velocity for the three systems in Table 2 for $t_t = 5$ mm. Solid line: NASA BLE; dashed line: model-I; dash-dotted line: model-II. $H^* = 4829.9$ BHN, with $k = 1.8$. Open circle corresponds to $\rho_p v_p^2 = E_t$ where the two models coincide.

Figure 1 compares d_c/t_t as a function of v_p/c_t for all three systems. Model-II follows the NASA BLE velocity dependence more closely, whereas model-I falls more steeply with velocity. The relative RMS deviations are 7.43% and 6.46% for models-I and -II, respectively.

4.2 Comparison with some experimental data

Next, we attempt to compare the model results and a few experimental datasets (Table 3) alongside the NASA BLE results. Perforation tests have been reported for several projectile materials, projectile diameters, and target materials (Fish and Summers, 1965; Clough et al., 1966, 1970; Hayduk et al., 1973; Hayashida and Robinson, 1991; Hörz, 2012).

Table 3: Data pertaining to perforation experiments used in Fig. 2. Sources: HR91, (Hayashida and Robinson, 1991); H12, (Hörz, 2012); H73, (Hayduk et al., 1973); C66, (Clough et al., 1966); C70, (Clough et al., 1970).

Projectile	Target	ρ_p (kg/m ³)	d (mm)	E_t (GPa)	ρ_t (kg/m ³)	$\tilde{H}_t$ (BHN)	v_p (km/s)	t_t (mm)	Source
Al 1100	6061-T6	2700	0.40	68.9	2700	95	6.78	1.27	HR91
Soda-lime glass	Al 1100-O	2500	3.18	69.0	2700	23	2.0	4.90	H12 ^a
							4.0	7.01	
							5.0	8.80	
							5.9	10.4	
							6.7	11.1	
Nylon	2024-T4	1200	1.59	73.2	2700	120	3.14	1.55	H73 ^b
							3.39	1.78	
							4.01	2.07	
							4.53	2.39	
							5.36	2.68	
Nylon	316 SS	1200	3.18	195	7800	150	7.46	4.32	C70 ^c
Pyrex	316 SS	2260	2.38	195	7800	150	7.62	4.60	C70 ^c
Pyrex	2024-T6	2260	2.38	70.4	2700	125	7.62	8.82	C66 ^c
Pyrex	Cb-1Zr	2260	3.18	110	8050	100	7.62	7.36	C70 ^c
Pyrex	Cb-1Zr	2260	2.38	110	8050	100	7.62	4.95	C70 ^c
Cadmium	2024-T3	8650	3.18	73.2	2700	120	8.0	23.7	H73 ^b
							8.4	25.4	

^aThickness at which the hole diameter vanishes. ^bLeak criterion. ^cVisible hole; thickness at incipient perforation.

To that end, we recast Eq. (14) as

$$\frac{t_t}{d} \left(\frac{H_t}{H^*} \right)^{1/4} = \Pi^\gamma, \quad \Pi = \frac{\rho_p v_p^2}{E_t}, \quad \gamma = \frac{2}{5} \text{ or } \frac{1}{3} \quad (15)$$

where Π is a Cauchy number, which is the ratio of the inertial stress of the projectile over the elastic stiffness of the target. Likewise, the NASA BLE (Eq. (2)) can be expressed as

$$\frac{t_t}{d} \left(\frac{H_t}{H^*} \right)^{1/4} = \frac{5.24 k d^{1/18}}{(H^*)^{1/4}} \left(\frac{\rho_t}{\rho_p} \right)^{1/3-m} \Pi^{1/3}, \quad (16)$$

with $k = 1.8$, d in cm, H^* in BHN, and m the density exponent[§] (cf. Eq. (2)).

Fig. 2 shows the experimental data and NASA BLE results superposed on the results from both models. It is useful to note that the experimental cases in Table 3 use different criteria to report perforation, including (i) locating the occurrence of the hole after impact by visual confirmation (Clough et al., 1966, 1970), (ii) checking leakage by pressurizing the target plate with helium (Hayduk et al., 1973), and (iii) critical target thickness at which the exit-hole on the target back face is not visible (Hörz, 2012). Overall, the models show reasonable corroboration with the experimental data for a wide range of Π . The largest difference is for the cadmium projectile, which has the highest density contrast with the aluminum

[§]Recall that in the NASA BLE $m = 1/2$ for $\rho_t/\rho_p > 0.67$ and $m = 2/3$ for $\rho_t/\rho_p < 0.67$.

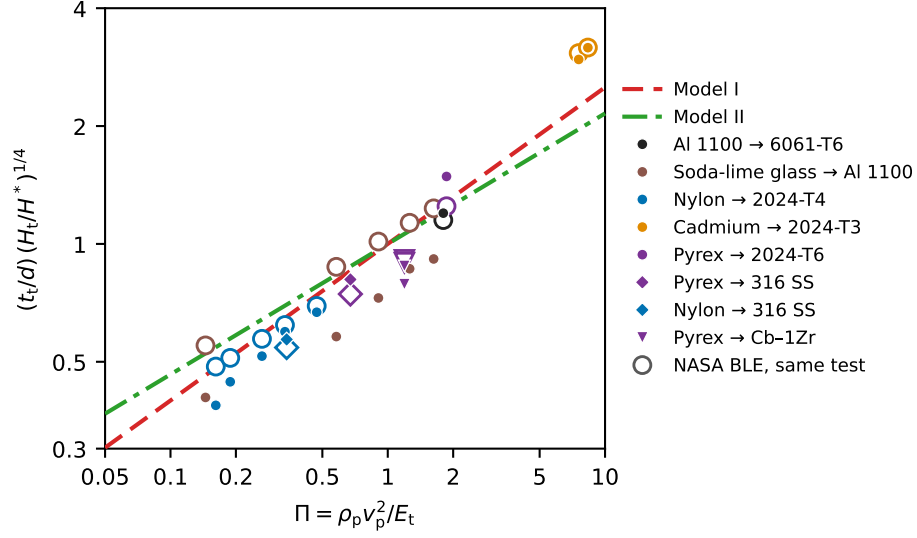


Figure 2: A comparison of the normalized target thickness $(t_t/d) (H_t/H^*)^{1/4}$ at the threshold of perforation versus Cauchy number ($\Pi = \rho_p v_p^2 / E_t$) predicted by the models (lines) and NASA BLE (open symbols) with experimental data (filled symbols, Table 3). Note that both axes are logarithmic.

target. Of all the experimental datasets shown in Fig. 2, only the cadmium–Al combinations ($\Pi = 7.56$ and 8.34) have $\rho_t/\rho_p < 0.67$, which translates to $m = 2/3$ in Eq. (16). That additional density ratio effect in the NASA BLE captures the experimental data well, unlike the models where the additional density effect is absent. Interestingly, the plate-bending model (model-II) tends to overpredict (respectively, underpredict) the threshold target thickness at $\Pi < 1$ (respectively, $\Pi > 1$) by a higher margin than the indentation model (model-I).

5 Summary and conclusions

This article examines how much of the structure of the NASA single-wall BLE follows from elementary mechanics and high-school physics. Energy balances for indentation and plate bending give simple power laws for the critical projectile diameter, which follow (very roughly) a structure similar to that of the BLE, which was based on the seminal work of Cour-Palais and others. Clearly, the Hertzian and plate-bending relations are strong idealizations. Nonlinear material behaviors combined with geometric nonlinearity, wave propagation, heating, melting, fragmentation dynamics, and ultimate fracture are critical. Likewise, the geometric condition defined as a failure threshold in both models is a rather simplistic choice borne out of convenience rather than a rigorous failure condition. The authors found incorporating the material strength using the physics of impact cratering informative from a learning standpoint. The exercise illustrates both the usefulness of elementary mechanics for interpreting an empirical equation and the limitations of the assumptions that make the analysis simple.

Usage of AI

The authors sought the assistance of Claude to collate the experimental data in Table 3. Python scripts for the calibration of H^* and plotting figures were written with the help of Claude.

A Calibration of H^*

The amplitude H^* in Eq. (14) is the only fitted quantity in the two models. It is determined as a single scalar value for both models and the three systems considered in Table 2, with $t_t = 5$ mm and $k = 1.8$. For target system s (Table 2), model i (=model-I or -II), and a given velocity ($2 \leq v_j \leq 8$) km/s of the projectile, Eq. (14) is expressed as

$$\left(\frac{d_c}{t_t}\right)_{sij} = qB_{sij}, \quad B_{sij} = H_{t,s}^{1/4} \left(\frac{\rho_{t,s}}{\rho_p}\right)^{\gamma_i} \left(\frac{c_{t,s}}{v_j}\right)^{\lambda_i} \quad (\text{A.1})$$

where $q = (H^*)^{-1/4}$. Let $y_{sj} = (d_c)_{\text{BLE}}/t_t$ denote the normalized diameter from the NASA BLE (Eq. (2)) for the same velocity. For calibration purposes, the velocity range is divided into 500 equally spaced points between 2 and 8 km/s. Minimizing the sum of squared differences, $\sum_{s,i,j} (qB_{sij} - y_{sj})^2$, gives

$$q^* = \frac{\sum_{s,i,j} B_{sij} y_{sj}}{\sum_{s,i,j} B_{sij}^2}, \quad H^* = (q^*)^{-4} \quad (\text{A.2})$$

Each NASA BLE value enters the sum twice, once for each model, so that a single amplitude is shared. This gives $H^* = 4829.9 \approx 4830$ BHN. We also performed a similar exercise by fitting the models separately to the NASA BLE. Details aside, that approach gives $H^* \approx 5700$ BHN for model-I and $H^* \approx 4000$ BHN for model-II.

Defining $e_{sij} = q^* B_{sij} / y_{sj} - 1$, the RMS deviation for system s and model i is the square root of the mean of e_{sij}^2 over the velocity range (divided into equally spaced grid points), and the maximum is the largest $|e_{sij}|$ over the same range. Table 4) collates the RMS and maximum deviations for the three systems. The relative RMS deviations with $H^* = 4830$ are 7.43% and 6.46% for models-I and -II, respectively.

Table 4: Relative deviations of the models from the NASA BLE, Eq. (2) for $2 \leq v_p \leq 8$ km/s. The maximum is the largest absolute relative deviation.

Target	RMS (%)		Maximum (%)	
	I	II	I	II
Al 6061-T6	8.19	3.50	20.30	6.32
Ti-6Al-4V	7.20	2.44	18.10	3.89
304L steel	6.84	10.35	12.28	11.96
Single Calibration	7.43	6.46	20.30	11.96

References

- Chen, X., 2005. Foreign object damage on the leading edge of a thin blade. *Mechanics of Materials* 37, 447–457.
- Christiansen, E.L., Arnold, J., Davis, A., Hyde, J., Lear, D., Liou, J.C., Lyons, F., Prior, T., Ratliff, M., Ryan, S., et al., 2009. Handbook for Designing MMOD Protection. Technical Report JSC-64399, Version A. NASA Johnson Space Center. Houston, TX.
- Clough, N., Lieblein, S., McMillan, A.R., 1970. Dimple, Spall, and Perforation Characteristics of Thin Plates of Nine Materials under Hypervelocity Impact. Technical Report NASA TN D-5625. NASA Lewis Research Center. Cleveland, OH.
- Clough, N., McMillan, A.R., Lieblein, S., 1966. Dimple, Spall, and Perforation Characteristics in Aluminum, Columbium, and Steel Plates under Hypervelocity Impact. Technical Report NASA TN D-3468. NASA Lewis Research Center. Cleveland, OH. NTRS document 19660018720.
- Cour-Palais, B.G., 1987. Hypervelocity impact in metals, glass and composites. *International Journal of Impact Engineering* 5, 221–237.
- Cour-Palais, B.G., Crews, J.L., 1990. A multi-shock concept for spacecraft shielding. *International Journal of Impact Engineering* 10, 135–146.
- Fish, R.H., Summers, J.L., 1965. The Effect of Material Properties on Threshold Penetration. Technical Report NASA TM X-54786. NASA Ames Research Center. Moffett Field, CA. Presented at the Seventh Hypervelocity Impact Symposium, Tampa, FL, November 1964.
- Hayashida, K.B., Robinson, J.H., 1991. Single Wall Penetration Equations. Technical Report NASA TM-103565. NASA Marshall Space Flight Center. Huntsville, AL.
- Hayduk, R.J., Gough, P.S., Alfaro-Bou, E., 1973. Correlation of New Hypervelocity Impact Data by Threshold Penetration Relations. Technical Report NASA TN D-7308. NASA Langley Research Center. Hampton, VA. NTRS document 19730022133.
- Holsapple, K.A., 1993. The scaling of impact processes in planetary sciences. *Annual Review of Earth and Planetary Sciences* 21, 333–373.
- Holsapple, K.A., Schmidt, R.M., 1987. Point source solutions and coupling parameters in cratering mechanics. *Journal of Geophysical Research: Solid Earth* 92, 6350–6376.
- Hörz, F., 2012. Cratering and penetration experiments in aluminum and Teflon: Implications for space-exposed surfaces. *Meteoritics & Planetary Science* 47, 763–797.
- Hörz, F., Cintala, M.J., Bernhard, R.P., Cardenas, F., Davidson, W.E., Haynes, G., See, T.H., Winkler, J.L., 1995. Penetration Experiments in Aluminum 1100 Targets Using Soda-Lime Glass Projectiles. Technical Report NASA TM-104813. NASA Johnson Space Center. Houston, TX.

- Johns, R., 1974. Fod impact testing of composite fan blades, in: Natl. SAMPE Symp. and Exhibition.
- Kessler, D.J., Cour-Palais, B.G., 1978. Collision frequency of artificial satellites: The creation of a debris belt. *Journal of Geophysical Research: Space Physics* 83, 2637–2646.
- Li, M., Ru, C.Q., Gao, C.F., 2018. Axisymmetric indentation of an elastic thin plate by a rigid sphere revisited. *ZAMM – Journal of Applied Mathematics and Mechanics* 98, 1436–1446.
- Rogers, J.A., Bass, N.T., Wiest, M.L., Wantz, Z., Wilkerson, J.W., Lacy Jr, T.E., 2024. The pursuit of hypervelocities: A review of two-stage light gas gun aeroballistic ranges. *International Journal of Impact Engineering* 185, 104861.
- Ryan, S., Christiansen, E.L., 2010. Micrometeoroid and Orbital Debris (MMOD) Shield Ballistic Limit Analysis Program. Technical Report NASA/TM-2009-214789. NASA Johnson Space Center. Houston, TX.
- Tabor, D., 2000. *The Hardness of Metals*. Oxford University Press, Oxford.
- Timoshenko, S., Woinowsky-Krieger, S., 1959. *Theory of Plates and Shells*. McGraw-Hill, New York. 2nd edition.
- Watts, A.J., Atkinson, D., 1995. Dimensional scaling for impact cratering and perforation. *International Journal of Impact Engineering* 17, 925–935.
- Watts, A.J., Atkinson, D., Rieco, S., 1993. *Dimensional Scaling for Impact Cratering and Perforation*. Technical Report NASA-CR-188259. POD Associates, Inc., for NASA. Albuquerque, NM.
- Whipple, F.L., 1947. Meteorites and space travel. *The Astronomical Journal* 52, 131.