

Finite Element Modeling of the Structural Response of a Reinforced Concrete (RC) Box Culvert with Edge Beam and Skewness

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ABSTRACT

Reinforced concrete (RC) box culverts are buried in structures installed under highways for various purposes, primarily for water flow. Usually, non-skewed culverts are preferred, but in areas with topography that require skewed culverts, cast-in-place RC culverts may be considered. Researchers have extensively investigated the structural performance of the non-skewed culverts. Limited research studies are available to understand the structural behavior of RC-skewed culverts. Therefore, this study examined the impact of a skew angle in the top slab on the structural response of an RC culvert provided with an edge beam. Finite element modeling (FEM) was employed to simulate RC culverts using a shell element model with varying skew angles of 0, 15, 30, 45, and 60 degrees in Abaqus. A solid model was introduced to confirm the shell model's limitation of capturing localized failure in the edge beam. It is observed that the culvert's edge beam with adequate shear reinforcement is essential to maintain its ultimate capacity. The study establishes a finite-element-based approach for evaluating load-deflection response in skewed culvert systems, supported by solid-element modeling to examine localized effects. The results provide a consistent basis for comparison across configurations, with emphasis on relative response trends rather than absolute prediction of ultimate capacity.

Keywords: Reinforced-concrete, skewed box culverts, Finite element modeling

1 INTRODUCTION

2 Box culverts are an essential infrastructure associated with transportation. These are provided
 3 under highways and railways for water passage. Box culverts are classified by installation
 4 (trench or embankment) and the material used. Cast-in-place RC box culverts are one type, and
 5 studies have been conducted to thoroughly assess their structural performance under potential
 6 loads throughout their lifespans. It mostly encounters dead loads from the backfill on its top slab
 7 and live loads from vehicles. Depending on site conditions, it may also be subject to water
 8 pressure, soil pressure, and seismic activity. Due to the critical nature of the potential dead and
 9 live loads, extensive studies based on experimental testing and finite element modeling (FEM)
 10 were conducted to understand their structural behavior. The contribution from the live load
 11 becomes more significant compared to dead loads in the case of a small depth of backfill on the
 12 culvert's top slab (Abdel-Karim et al. [1]). Orton et al. [2] proposed an optimum fill depth of 6ft.
 13 After that, the effects of live load on the culverts can be ignored. He further confirmed that live
 14 load can be ignored when the live load pressure is less than 10% of the dead load on the culvert.
 15 Liu et al. [3] quantified the risk of early-age cracking in cast-in-place culverts through a
 16 parametric study that considered several factors, including coefficient of thermal expansion,
 17 construction sequence, and concrete type, using a validated FEM methodology in Abaqus. Ulger
 18 et al. [4] conducted field and FEM studies to optimize culvert load ratings and concluded that
 19 load ratings based on 2D models and AASHTO provisions are overly conservative. In contrast,
 20 3D models had some good agreement with field observations. Moradi et al. [5] studied the
 21 impact of rigid and flexible pavements on the load distribution on culverts with low-depth soil
 22 fill by carrying out an experimental and numerical study. He concluded that the AASHTO LRFD
 23 overpredicts the effects of vertical live loads at low fill depths because it does not account for the
 24 additional load distribution provided by pavements. Woo et al. [6] investigated the effect of
 25 backfill on an inverted U-shaped RC culvert's ultimate capacity and fatigue life. Evan et al. [7]
 26 carried out experimental work to understand the fracture behavior of box culverts. The formation
 27 of cracks under increasing load was analyzed, and their widths were quantitatively measured.
 28 The structural damage index for a culvert was divided into five stages, and the corresponding
 29 crack width was proposed for each stage. Evan C. et al. [8] aimed to understand the shear
 30 strength of reinforced concrete box culverts, where size, reinforcement, and depth were critical
 31 factors. The specimens with greater steel content and deeper depths failed by shear, whereas
 32 those with less steel and shallower depths failed by flexure. Kabir, A. et al. [9] investigated the
 33 effect of reinforcement layout arrangements on the ultimate capacity of skew slabs. Four slabs
 34 were tested to failure with different rebar layouts, and a Finite Element Mindlin plate
 35 formulation was used to study their numerical response. Bugher, C et al. [10] evaluated the soil-
 36 culvert interaction using 3-D FEM models with specialized infinite elements and found that these
 37 elements significantly reduced computational cost while retaining accuracy. James H. et al [11]
 38 employed finite element modeling to predict the ultimate capacity of the skew culvert and the
 39 artificial neural network (ANN) to develop simple mathematical equations. A structural model
 40 was developed in ANSYS to simulate the performance of a single-barreled reinforced concrete
 41 culvert. Non-linear constitutive models were utilized to capture the post-yielding behavior of
 42 concrete and steel. Regression analysis was performed on previously collected experimental data
 43 to develop a mathematical equation for the compressive stress-strain relationship of concrete,
 44 which was then incorporated into the FE model. Gong, Y. et al. [12] performed experimental and
 45 numerical simulations on reinforced concrete box culvert (BC) failure. Four RC box culverts,
 46 including integral BC, round-hinged BC, flat-seam BC, and mortise BC, were designed and

1 tested. It was concluded that integral BC has the maximum carrying capacity, and the failure
2 mode was the tensile failure of concrete at the bottom of the top slab. FEM models were also
3 validated and studied for mechanical response.

4 **RESEARCH SIGNIFICANCE**

5 Finite element modeling (FEM) is the most widely used numerical technique for simulating
6 complex phenomena in nature by solving differential equations. Researchers usually employ
7 parametric studies to expand their knowledge and approach data, which, in turn, helps
8 practitioners make well-informed design decisions. Due to limited experimentation and design
9 guidelines for complex structural geometries, failing to employ an FEM study may result in an
10 uneconomical, conservative, and sometimes unsafe design. Therefore, by developing a simplified
11 shell element model, this study evaluates the impact of skew angle on the flexural response of an
12 RC box culvert designed according to AASHTO LRFD and having an integrated edge beam.
13 Moreover, this study highlights the comparison behavior of the shell and solid element models
14 for these large structures.

15 **FINITE ELEMENT MODELING**

16 Finite Element Modeling (FEM) is one of the widely accepted numerical techniques to solve
17 complex structural engineering problems. Solving 3-D problems with 2-D FE techniques (plane
18 stress, plane strain) is widely recognized as cost-effective for computational efforts. However,
19 converting a real 3-D problem into a 2-D FE solution is not always a choice for researchers due
20 to the complex geometry of a structure. In the present study, the skew angle in the geometry of an
21 RC culvert and the edge beam provision have ruled out any choice but the 3D FE model.
22 Additionally, researchers have increasingly used 3D models for decades to visualize stress
23 distributions throughout the structure better, driven by increases in computing power. Reliable
24 results that inform design decisions can justify the computational cost of 3D FE models.

25 **MATERIAL MODEL**

26 In this study, the concrete damage plasticity (CDP) model, built into the commercial FE code
27 'Abaqus', was adopted to simulate the material response of the culvert. The CDP has vast
28 popularity and confidence amongst investigators regarding its ability to predict the structural
29 response of any RC structure. The model is a continuum, plasticity-based damage model for
30 concrete. It assumes two failure mechanisms: tensile cracking and concrete crushing. The rebars
31 can be defined independently of the concrete, and the associated effects, such as dowel and slip
32 action, can be incorporated into the concrete model through tension-stiffening parameters to
33 simulate load transfer across cracks via the rebars. [Abacus Documentation]. The typical non-
34 linear stress-strain data for the rebars utilized in the study were generated based on the material
35 yield strength. While the concrete has a compressive strength of 5000 psi, the non-linear stress-
36 strain data for compressive behavior and the bi-linear stress-relative displacement data for tensile
37 behavior were incorporated in accordance with CEB-FIP-2010.

38 **ANALYSIS PROCEDURE**

39 An implicit analysis can be conducted to predict the structure's response. Still, this analysis faces
40 challenges in terms of convergence, especially for these large reinforced concrete box culverts.
41 Therefore, dynamic explicit analysis is preferably adopted in the present study. The dynamic
42 explicit procedure determines the response state at the end of the time increment based on the
43 displacements, velocities, and accelerations at the beginning of the increment. Therefore, a small
44 time increment is required to obtain stable, accurate results, but this comes at the cost of a large

number of time increments. Moreover, a quasi-static condition can also be achieved in dynamic explicit if the loads are applied slowly. Accordingly, the dynamic explicit analysis procedure is conducted in this current study.

FE MODELSIMPLIFICATIONS

This work investigates the impact of skewness in the top slab on the load-deflection response of an RC culvert with an edge beam under simulated vehicle loading. Therefore, ignoring parameters such as the bottom slab and the backfill soil on the top slab and against the walls is quite rational.

The following is the FE model simplification summary for the worst loading scenario on the top slab.

1. The backfill is ignored.
2. The bottom slab is ignored while fixed boundary conditions are ensured, which makes the conditions almost identical. Additionally, one FE model analysis with a bottom slab with appropriate boundary conditions was performed to observe its impact on the response. (Figure 1)
3. 6 feet of the base of the non-skew culvert is considered to ensure there is no in-plane movement of the culvert during the loading. The RC culvert slab is designed as a one-way slab (2D frame analysis required) using the unit-width method per AASHTO LRFD provisions. Moreover, the skew culverts have an additional length to accommodate the specific skew angle. Furthermore, one FE model is run with symmetric boundary conditions along the length of the non-skew model to ensure there's no significant influence on the response. (Figure 2)
4. The culvert's extended skew portion reinforcement follows the same rebar schedule to maintain a uniform basis for comparison.
5. Main reinforcement in the skew portion is provided perpendicular to the side walls.

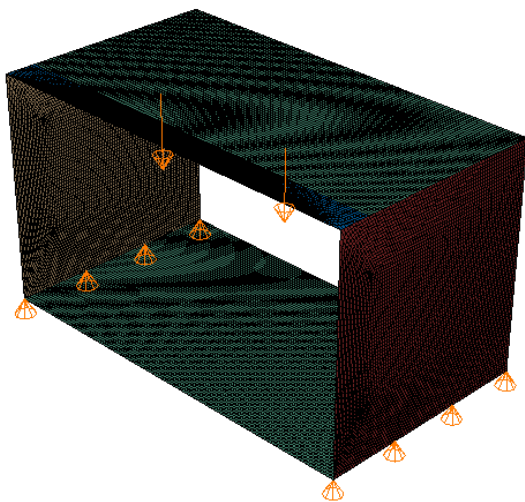


Figure 1: Shell Model with Bottom slab (0-degree)

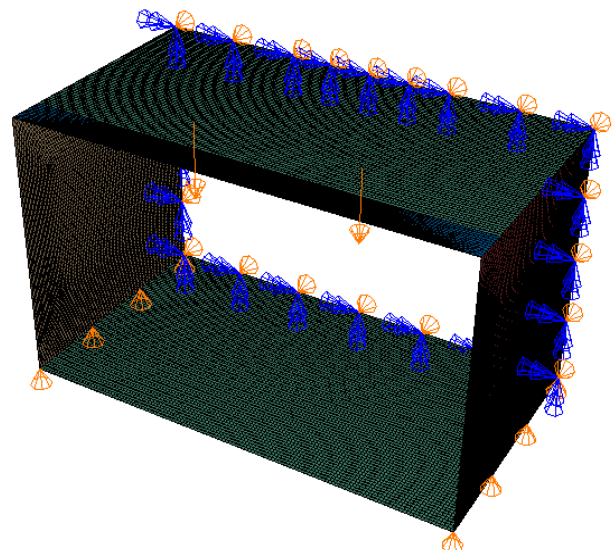


Figure 2: Shell Model (SYMM along length)

SHELL ELEMENT MODEL

These 10ft span and 6ft height culvert models used a 4-node doubly curved shell, which can be thin or thick, with reduced integration and hourglass control, along with a finite membrane strain element. S4R is an element that has six degrees of freedom: three translations and three rotations. Reduced integration helps lower computational costs and prevents shear locking in bending-dominated problems. The shell models used nine integration points at the section level to accurately capture the section response. The shell element was assumed to be at the mid-sections of the slab, beam, and walls, with thicknesses defined by the culvert geometry (10 in for the slab, 8 in for the walls, and 23 in for the edge beam). Reinforcements were added at specific locations in the slab, walls, and beam, aligned with the design. It's important to note that reinforcements are smeared into the shell element, assuming their uniform distribution over

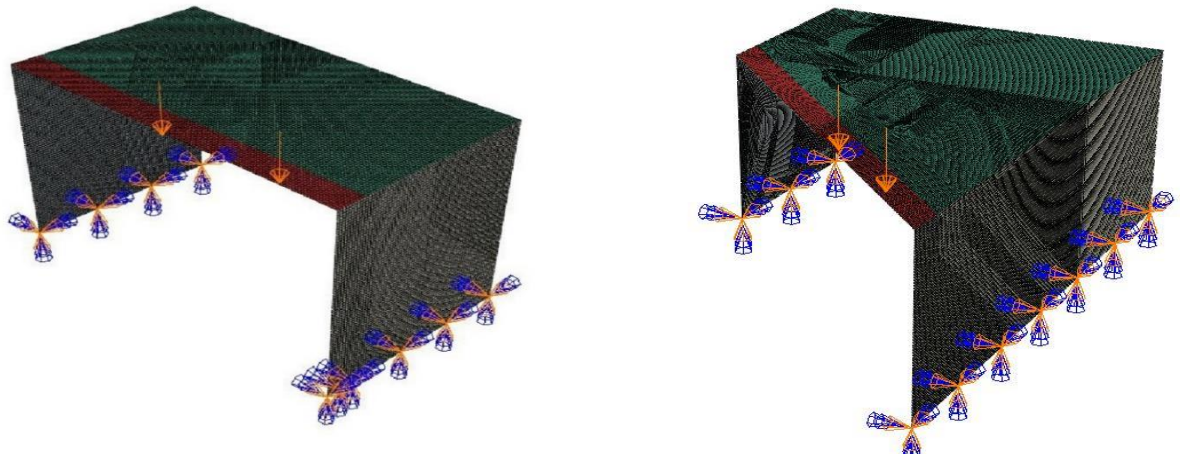


Figure 3: Shell Model (0-skew and 45-skew)

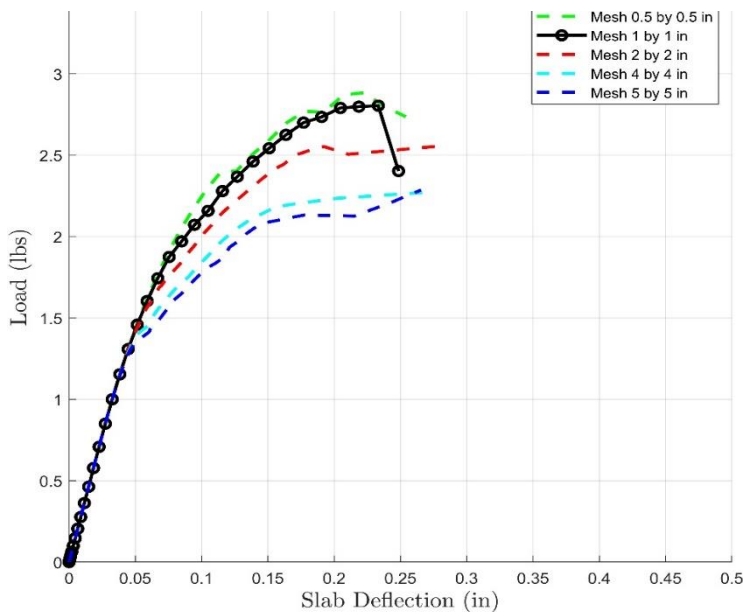


Figure 4: Mesh Study (0-degree Shell Model)

the concrete element along a designated orientation.

An adopted shell element methodology comprises an edge beam (Figure 3: Red region) at one end of the culvert. The thickness of the edge beam is greater than the slab thickness, and the beam reinforcement is defined accordingly. For simplicity, the haunches were ignored in the models. A mesh density study (Figure 4) is conducted, and an overall 1 in by 1 in mesh size is selected for further analysis. Loadings were applied at specific nodes, spaced 4ft apart, to simulate axle loading, and all rotational and translational degrees of freedom were restricted at the bottom to simulate a fixed boundary condition. Explicit analysis was utilized, considering its computational efficiency and

fewer convergence problems for an indeterminate structure in a way that ensures the noninvolvement of any movement in the structural response based on inertia.

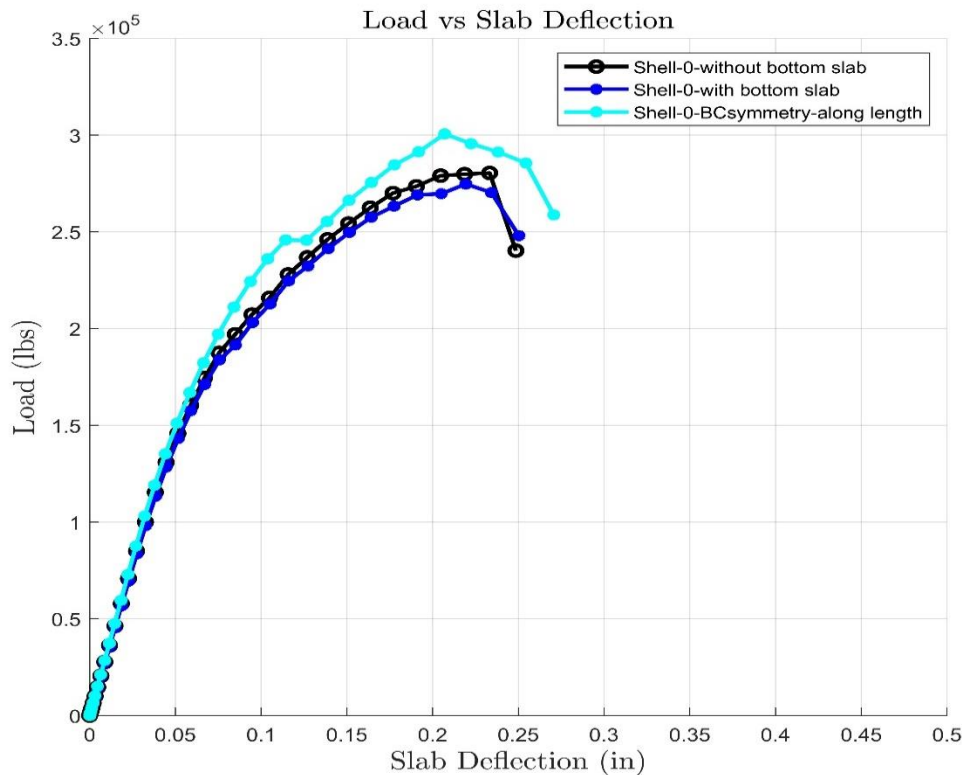


Figure 5: Load-Deflection Graph (Non-Skew Model)

SOLID ELEMENT MODEL

Creating a solid element model (Figure 6) is more laborious than a shell element model in terms of complexity and computational efforts (defining rebars, number of nodes, postprocessing), but in the present study, it has an evident influence on understanding the response depicted by the shell model and verifying the limitation involved. The non-skew solid (2 in by 2 in) element model was utilized (Figure 7). For this purpose, an 8-node linear brick, reduced-integration,

hourglass-control solid element was utilized. The hexahedral element was used because it yields accurate results for large deformations, whereas the tetrahedral element typically provides a stiffer response. The 2-node linear 3D beam element rebar was embedded in the host 'concrete' using Abaqus's embedded region feature. The loadings were applied to the nodes using 10" by 20" steel plates. All six translations were restricted at the bottom nodes to simulate the fixed boundary conditions. For the skewed cases, a relatively coarser mesh was used for walls to reduce computational cost, which is adequate for capturing load-deflection trends but not intended for detailed local stress evaluation.



Figure 6: Solid Model (Non-Skew)

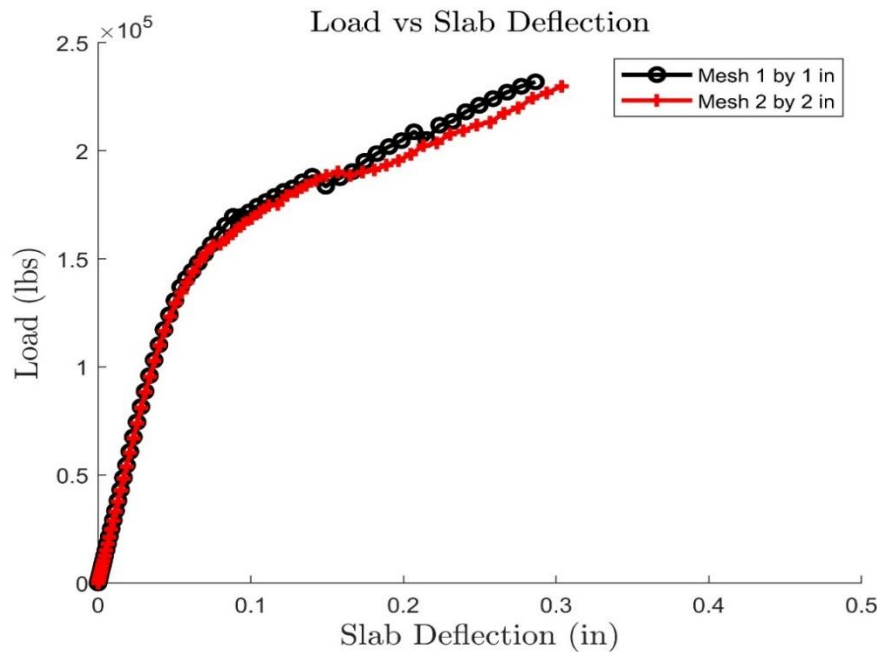


Figure 7: Mesh Study (0-degree Solid Model)

1 RESULTS AND DISCUSSION

2 The present study aims to develop a simplified FE model that can simulate the flexural response
 3 of an RC culvert with an edge beam and a skew angle. For this purpose, the shell element-based
 4 model is adopted to capture the structural response of a non-skew RC culvert (Figure 9a).
 5 Figure 8 shows that the load-deflection graphs for both the FE shell and solid models up to
 6 maximum tandem load (50000 lbs) are in complete agreement. At the same time, those for up to
 7 150 kips are in acceptable agreement and demonstrate the FE shell model's validity with respect
 8 to boundary conditions, loading, and overall geometric stiffness. However, it is noted that the
 9 shell model has unique limitations in accounting for the edge beam's integrated role with the
 10 culvert's slab. The shell model can capture the overall flexural behavior but cannot simulate the
 11 localized shear behavior of an edge beam. Figure 8 shows that, after 150 kips, the load-deflection
 12 behavior of the solid.

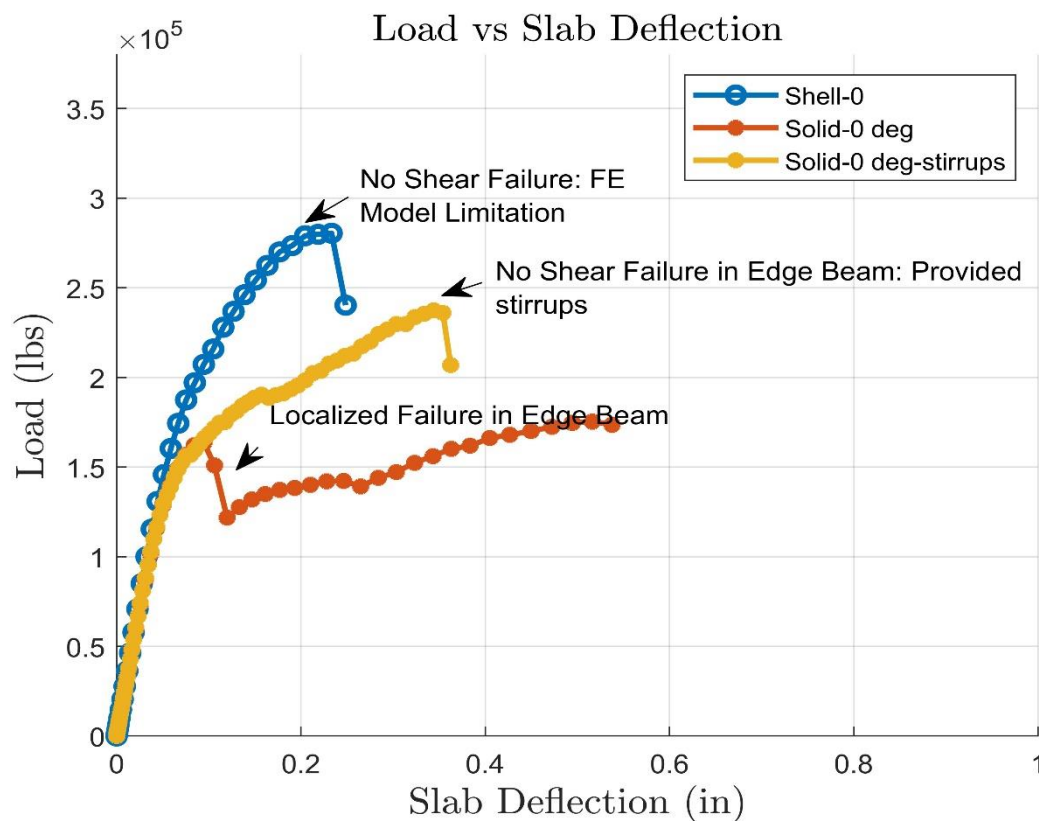


Figure 8: Load-deflection study of Comparative FE Models

24 model
 25 without stirrups in the edge beam deviates from the other comparative FE models, as it shows
 26 severe localized cracking due to stress concentrations (Figure 9b) that progressively span along
 27 the length of the edge beam, resulting from the lack of shear reinforcement. A solid element
 28 model was employed to qualitatively confirm this hypothesis regarding the shell model's
 29 inherent inability to capture localized shear failure in the edge beam. The solid element model is
 30 volume-element-based and is inherently capable of capturing the flexural and local shear
 31 responses of these integrated geometries, comprising two different structural components (a slab
 32 and an edge beam).

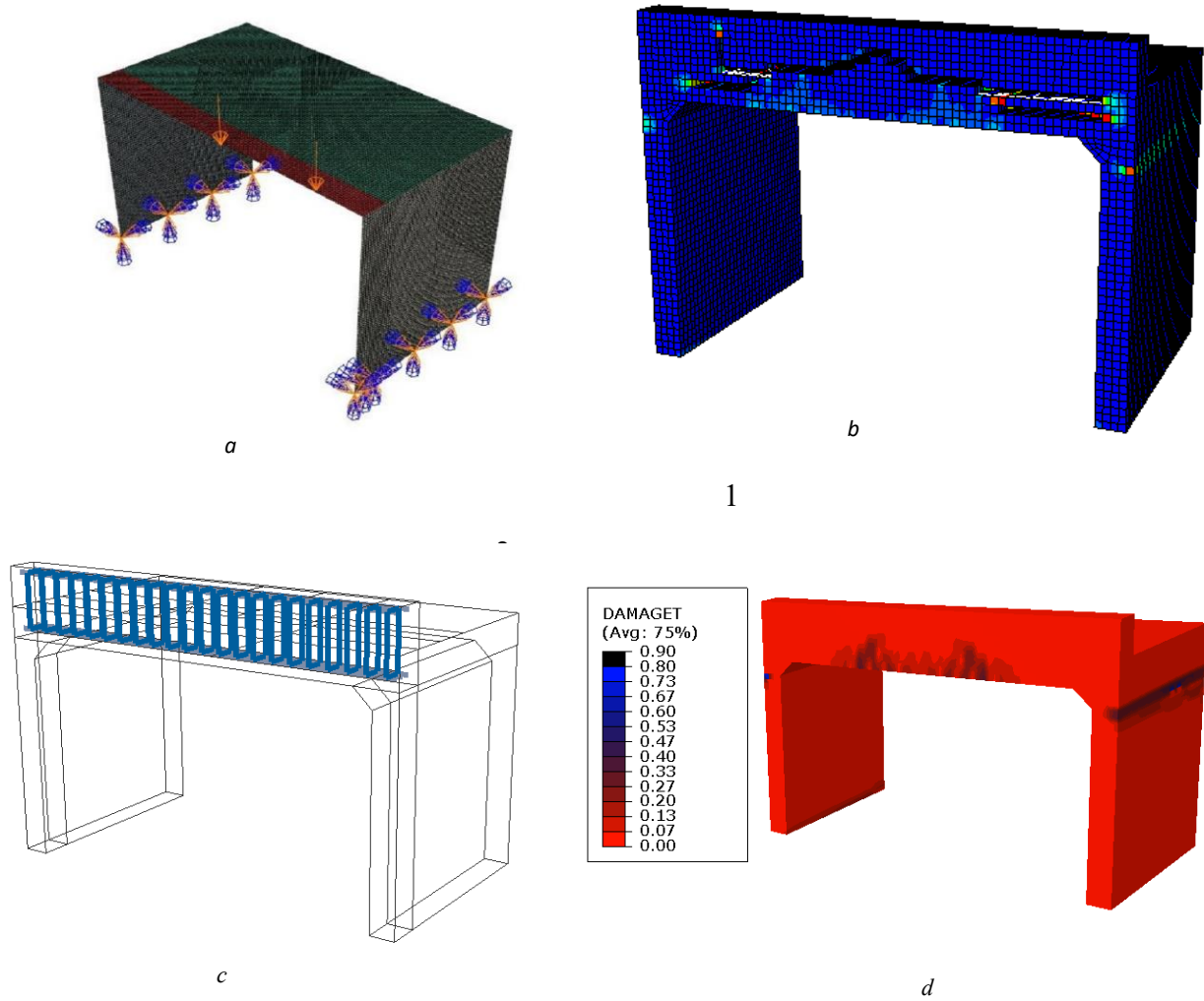


Figure 9: a: Shell Model, b: Solid Model with localized failure in Edge Beam, c: Solid Model with Stirrups, d: DAMAGET near Ultimate Loading (Flexural Cracking in Edge Beam)

In the solid model (Figure 9c), #4@6in c/c double-leg stirrups are provided to control localized shear failure at the edge beam while achieving a flexurally controlled response (Figure 9d). By providing shear reinforcement, one can compare the solid model response with the shell model response and further confirm the hypothesis discussed above. Figure 8 shows that the shell and solid model responses are in good agreement, up to 150 kips. After that, the solid model shows a less stiff behavior, reaching an ultimate load of 235 kips at 0.35 in displacement, compared to the shell model, which reaches 280 kips at 0.23 in displacement, indicating 20% less ultimate capacity.

The above difference in the stiffness response at the later load stages may be because of the shell element's inbuilt nature to incorporate the reinforcement in a smeared form, which means the uniform distribution of rebar stiffness throughout the elements. Meanwhile, the rebars are provided discretely in the solid element. Therefore, the FE shell model captures the upper-bound flexural behavior (20% higher) of an RC culvert with an edge beam. This upper-bound

effect does not contradict the purpose of the later parametric study, which aims to change only the skew angle in the top slab and observe the overall effect on global resistance behavior.

The localized cracking observed near the edge beam reflects stress concentration and initiation of localized damage at the boundary. These observations are presented for qualitative understanding of the response and are not intended as a detailed assessment of shear failure behavior.

Therefore, the discussion is limited to global response characteristics.

SHELL–SOLID MODEL COMPARISON FOR SKEWED CULVERT CONFIGURATIONS

Figure 10-12 shows that the load–deflection responses obtained from the shell and solid models with shear rebars in the edge beam remain consistent in overall trend for both 30° and 60° skew configurations, indicating that the primary structural behavior is adequately captured. For the 30° skew configuration with shear reinforcement provided in the edge beam, the solid model response remains stable. It shows acceptable agreement with the corresponding shell model, with

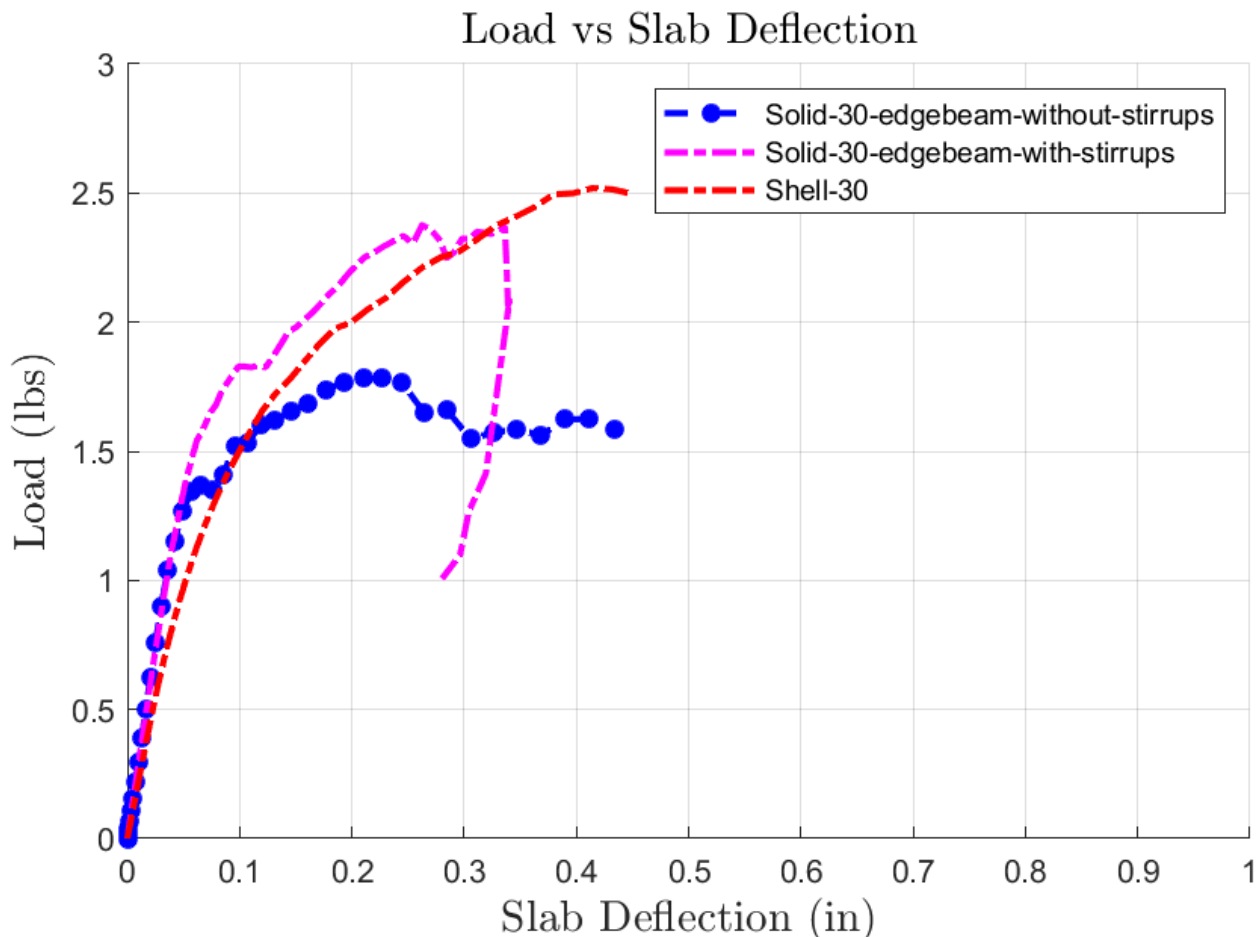


Figure 10: Shell-Solid Model Comparison for 30-degree

only minor differences in stiffness. However, a slight difference in stiffness is observed in the 60° skew case at load stages earlier than 50000 lbs, where the solid model exhibits a slightly steeper slope than the shell model. This deviation may be attributed to the greater influence of

- 1 three-dimensional stress states and torsional effects at higher skew angles, which are more
- 2 explicitly accounted for in the solid formulation.
- 3 For the 30° and 60° skew configurations without shear reinforcement in the edge beam, localized

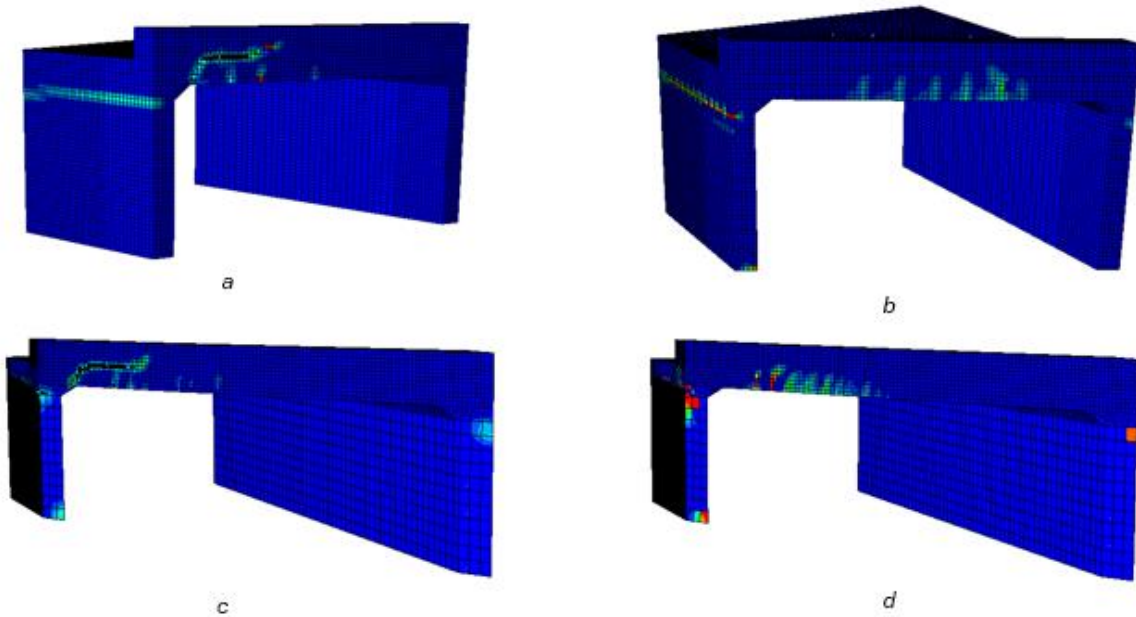


Figure 11: a: Solid-30-without-Stirrups in Edge beam, b: Solid-30-with Stirrups in Edge beam, c: Solid-60-without-Stirrups in Edge beam, d: Solid-60-with-Stirrups in Edge beam

- 4 damage is observed in the solid model and is qualitatively confirmed by the corresponding
- 5 damage patterns. However, this local failure is not reflected in the load–deflection response, as
- 6 no abrupt stiffness degradation or load drop is captured. Furthermore, the 30 ° solid model
- 7 without stirrups exhibits irregular behavior beyond approximately 150 kips, attributed to the
- 8 explicit analysis approach and associated numerical instability following damage initiation.
- 9 Therefore, the post-150-kip response for this case is not considered reliable for quantitative
- 10 interpretation.

Despite these differences, the agreement in overall response behavior confirms that the shell model captures the governing mechanics of the culvert with skewness and edge beam. Therefore, shell formulation is considered appropriate for parametric study, where the objective is to evaluate trends in stiffness rather than to resolve localized three-dimensional effects.

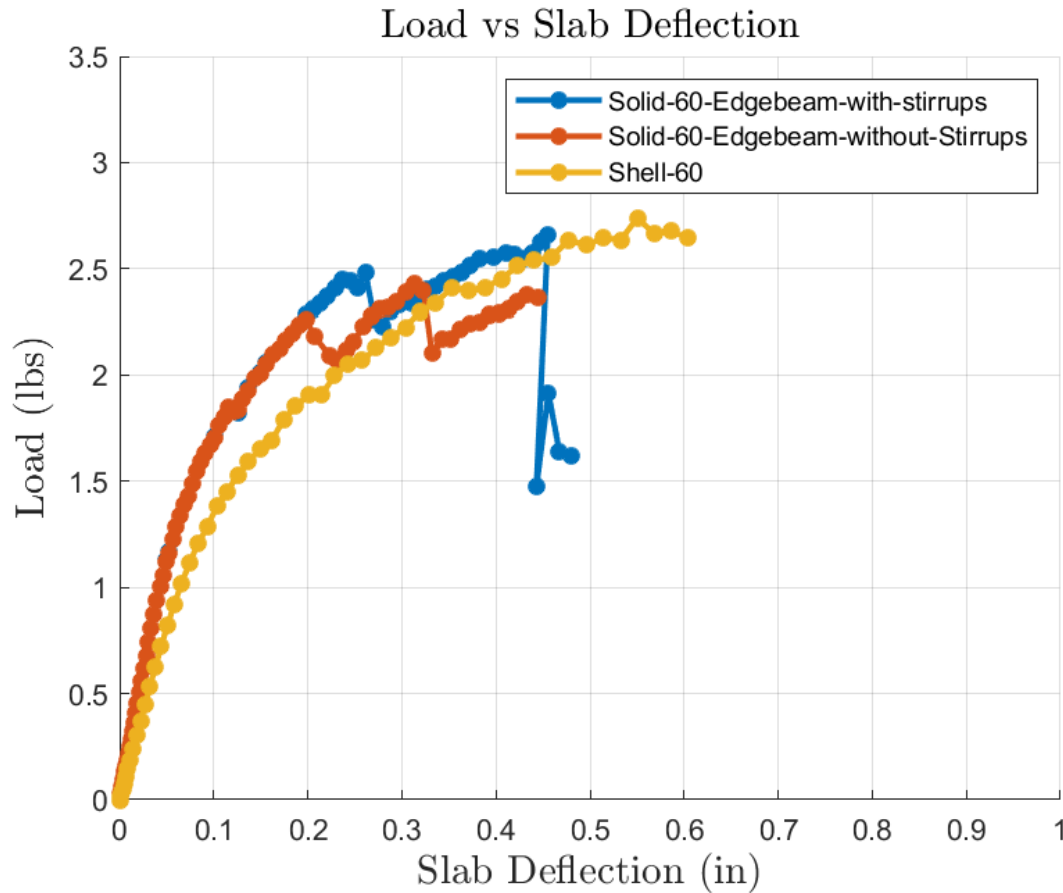


Figure 12: Shell-Solid Model Comparison for 60-degree

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EDGE BEAM RESPONSE AND DESIGN CONSIDERATION

DOTs typically adhere to their standard field/design procedures, especially regarding the edge beam of an RC culvert. The standard edge beam design with stirrups is also available in the Erikson software, which is often used for culvert design. Conversely, a typical beam design may lack double-angle stirrups to mitigate localized shear failure at higher loads. Although the solid model shows the opposite at load levels above the maximum tandem load, it was expected that the edge beam, which is 23 inches deep by 10 inches wide and has only flexural reinforcement, could withstand localized shear failure. It concludes that shear reinforcement is also required in addition to the specified edge beam depth for the RC culvert to avoid any localized shear failure.

PARAMETRIC STUDY

The shell element model has been used in parametric studies (Figure 13) due to its robustness and fewer convergence issues, particularly for skewed RC culverts. On the other hand, because of the detailed rebars in each skew culvert, a solid element model incurs higher computational costs and more convergence issues. While the alternative shell element model can represent the overall flexural response, researchers may prefer not to use a solid model for these kinds of parametric studies. Moreover, in design methodologies, it is generally desirable to rely conservatively on the flexural response and avoid the structure's shear-control behavior. Therefore, the parametric investigation to determine the comparative flexural behavior of different skew culverts is not hampered by the shell model's inability to capture the localized shear failure in the edge beam. It is logical to expect that the edge beam will contribute to the flexural strength of an RC culvert and that through-thickness stresses, such as shear stress in the edge beam, will not be the governing factor for the load-deflection response in the case of the provision of proper shear reinforcement. In addition to the length needed for the skew portion of the culvert, all skew models with 15, 30, 45, and 60 degrees have the same 6 feet of base length. The displacement is measured at the center of the load points after the two concentrated loads are applied in the skew slab area, one foot away from the edge beam. The 0-skew culvert reaches a capacity of 280 kips at approximately 0.23 in slab displacement, as shown in Figure 12. The 15° skew culvert shows no noticeable effect of skew angle on the response up to the design tandem

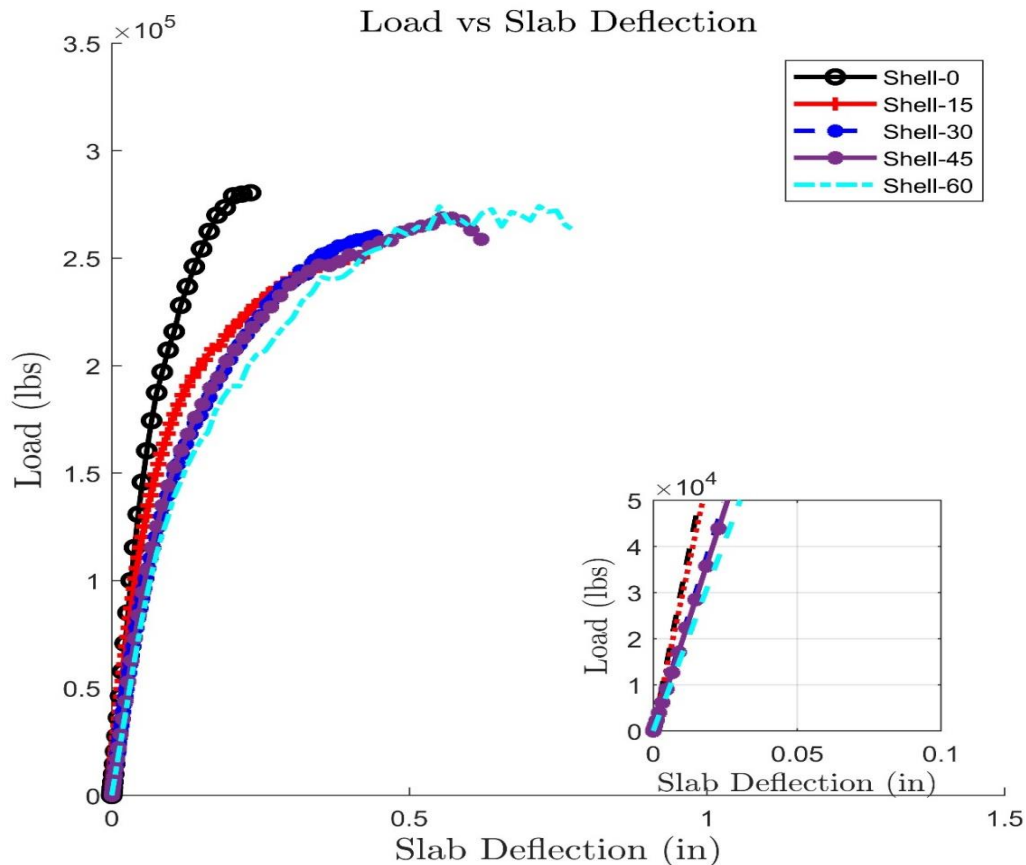


Figure 13: Parametric Study-Load vs Slab Deflection

load; however, beyond this point, it exhibits slightly less rigid behavior and reaches an ultimate capacity of 250 kips at 0.42 in.

The 30° skew case exhibits a marginally less rigid response compared to the 0° and 15° cases, even within the design load range, and attains a maximum capacity of 260 kips at 0.44 in. Similarly, the 45° skew model reaches a capacity of 265 kips at a displacement of 0.58 in, following a comparable trend. The minor shift in response observed for the 30° and 45° skew culverts can be attributed primarily to the slab's geometrical skew.

The 60° skew culvert reaches a capacity of approximately 265 kips at 0.50 in deflection, which is comparable to the 45° case, although the response is slightly less rigid. Based on the overall load–deflection behavior, it can be concluded that the current design approach for non-skew RC culverts remains applicable to skewed configurations.

The study focuses on evaluating load-deflection response trends across skewed culvert configurations within a consistent finite-element framework. The results are therefore interpreted comparatively rather than as absolute predictions.

CONCLUSIONS

1. Both non-skew and skewed culvert configurations demonstrated ultimate strengths at least three times the maximum tandem load (50,000 lbs), provided that no localized failure occurred. While these values indicate substantial reserve capacity, they are intended to represent comparative behavior across configurations, with greater confidence associated with the response within the service-level range.
2. The reinforcement detailing designed for a non-skew RC box culvert was found to be adequate for skewed configurations.
3. The skew angle in the top slab does not significantly influence the global flexural response of the culvert, if localized failure in the edge beam is prevented.
4. Adequate shear reinforcement in the edge beam is essential to ensure stable structural response and to achieve the full-strength capacity of the culvert system.
5. Comparison between shell and solid finite element models indicates that the shell formulation captures the overall structural behavior with reasonable accuracy. While localized damage in the skewed solid model (without shear reinforcement) can be qualitatively identified in the edge beam, it is not consistently reflected in the global load-deflection response. In contrast, the solid model reinforced with stirrups in the edge beam shows a stable response with good agreement with the shell model, supporting the use of shell elements for parametric evaluation of global behavior.

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AUTHOR CONTRIBUTION

The author contributed to conceptualization, finite element analysis, and Write-up.

REFERENCES

1. Abdel-Karim, A. M., Tadros, M. K., Benak, J. V., University of Nebraska, & ASCE. (1993). Structural response of Full-Scale Concrete Box culvert. In ASCE, *Journal of Structural Engineering* (Vols. 119–119, Issue 11, pp. 3238–3254). [https://ascelibrary.org/doi/10.1061/\(ASCE\)0733-9445\(1993\)119:11\(3238\)](https://ascelibrary.org/doi/10.1061/(ASCE)0733-9445(1993)119:11(3238))
2. Orton, S. L., Loehr, J. E., Boeckmann, A., & Havens, G. (2015). Live-Load Effect in Reinforced Concrete Box Culverts under Soil Fill. *Journal of Bridge Engineering*, [https://doi.org/10.1061/\(ASCE\)](https://doi.org/10.1061/(ASCE))
3. Liu, Y., Schindler, A. K., & Davidson, J. S. (2018). Finite-Element modeling and analysis of Early-Age cracking risk of Cast-In-Place concrete culverts. *Transportation Research Record Journal of the Transportation Research Board*, 2672(27), 24–36. <https://doi.org/10.1177/0361198118774157>
4. Acharya, R., Han, J., Brennan, J. J., Parsons, R. L., & Khatri, D. K. (2014). Structural Response of a Low-Fill Box Culvert under Static and Traffic Loading. *Journal of Performance of Constructed Facilities*, 30(1). [https://doi.org/10.1061/\(ASCE\)cf.1943-5509.0000690](https://doi.org/10.1061/(ASCE)cf.1943-5509.0000690)
5. Ulger, T., Okeil, A. M., & Elshoura, A. (2020). Load testing and rating of Cast-in-Place Concrete Box culverts. *Journal of Performance of Constructed Facilities*, 34(2). [https://doi.org/10.1061/\(ASCE\)cf.1943-5509.0001401](https://doi.org/10.1061/(ASCE)cf.1943-5509.0001401)
6. Moradi, M., Valipour, H., & Foster, S. (2015). Reserve of strength in inverted U-shaped RC culverts: effect of backfill on ultimate load capacity and fatigue life. *Journal of Bridge Engineering*, American Society of Civil Engineers. [https://doi.org/10.1061/\(ASCE\)BE.1943-5592.0000800](https://doi.org/10.1061/(ASCE)BE.1943-5592.0000800)
7. S.K. Woo, J.H. Jo, Y.G. Kwon (2010). Failure behavior characteristics of box culvert using 3-Axes loading system, *Fracture Mechanics of Concrete and Concrete Structures-High Performance, Fiber Reinforced Concrete, Special Loadings and Structural Applications*, 2010 Korea Concrete Institute, ISBN 978-89-5708-182-2
8. Evan C. Bentz, Richard Yee, and Michael P. Collins (2007). Investigating the Shear Strength of Concrete Box Culverts. Presented at the 2007 Transportation Research Board Annual Meeting, 2006.
9. Kabir, A., S. M. Nizamud-Doulah, M Kamruzzaman (2002). Bangladesh University of Engineering & Technology, Nizamud-Doulah, S. M., Bangladesh Institute of Technology, Kamruzzaman, M., & Bangladesh Institute of Technology. (2002). Effective reinforcement layout for skew slabs. *27th Conference on OUR WORLD IN CONCRETE & STRUCTURES*.
10. Bugher, C., Manahiloh, K. N., Kaliakin, V. N., & Shenton, H. W. (n.d.). Three-dimensional finite element analysis of reinforced concrete box culverts using infinite elements. In *Geo-Congress 2019: Vol. GSP 310* (pp. 193–194). ASCE
11. James H. Haido, Bashar A. Mahmood, Ayad A. Abdul-Razzak, Michael Dorn, and Salim T. Yousif (2020). PERFORMANCE OF RC SKEW BOX CULVERT: APPLICATION OF THE FINITE ELEMENT MODELLING AND ARTIFICIAL INTELLIGENCE. (2020). In *Journal of the University of Duhok: Vol 23* (Issue No.2)
12. Gong, Y., Ma, Y., Tan, G., Bi, H., Pang, Y., & Ma, C. (2020). Experimental study and numerical simulation on the failure process of reinforced concrete box culvert. *Advances in Civil Engineering*